

# Baryonium $X(1835)$

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The relativistic six-quark equations including the  $u$ ,  $d$  quarks and antiquarks are found. The nonstrange baryonia  $B\bar{B}$  are constructed without the mixing of the quarks and antiquarks. The relativistic six-quark amplitudes of the baryonia are calculated. The poles of these amplitudes determine the masses of baryonia. 16 masses of baryonia are predicted.

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## I. INTRODUCTION.

BES Collaboration observed a significant threshold enhancement of  $p\bar{p}$  mass spectrum in the radiative decay  $J/\psi \rightarrow \gamma p\bar{p}$  [1]. Recently BES Collaboration reported the results on  $X(1835)$  in the  $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$  channel [2, 3]. Under the strong assumption that the  $p\bar{p}$  threshold enhancement and  $X(1835)$  are the same resonance, Zhu and Gao suggested  $X(1835)$  could be a  $J^{PC} = 0^{-+} I^G = 0^+ p\bar{p}$  baryonium [4].

Theoretical work speculated many possibilities for the enhancement such as the  $t$ -channel pion exchange, some kind of threshold kinematical effects, as new resonance below threshold or  $p\bar{p}$  bound state [5–12].

In a series of papers [13–17] a method has been developed which is convenient for analysing relativistic three-hadron systems. The physics of the three-hadron system can be described by means of a pair interaction between the particles. There are three isobar channels, each of which consists of a two-particle isobar and the third particle. The presence of the isobar representation together with the condition of unitarity in the pair energies and of analyticity leads to a system of integral equations in a single variable. Their solution makes it possible to describe the interaction of the produced particles in three-hadron systems.

In our papers [18–20] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting quarks. The mass spectrum of  $S$ -wave baryons including  $u$ ,  $d$ ,  $s$  quarks was calculated by a method based on isolating the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered such approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions of the subenergy variables (as compared with the singular part of the amplitude) in the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In the previous paper [21] the relativistic six-quark equations are found in the framework of coupled-channel formalism. The dynamical mixing between the subamplitudes of hexaquark are considered. The six-quark amplitudes of dibaryons are calculated. The poles of these amplitudes determine the masses of dibaryons. We calculated the contribution of six-quark subamplitudes to the hexaquark amplitudes.

In the present paper the relativistic six-quark equations including  $u$ ,  $d$  quarks and antiquarks are found. The nonstrange barionia  $B\bar{B}$  are constructed without the mixing of the quarks and antiquarks. The relativistic six-quark amplitudes of the baryonia are calculated. The poles of these amplitudes determine the masses of baryonia. In Sec. II the six-quark amplitudes of baryonia are constructed. The dynamical mixing between the subamplitudes of baryonia is considered. The relativistic six-quark equations are obtained in the form of the dispersion relations over the two-body subenergy. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. Sec. III is devoted to the calculation results for the baryonia mass spectrum and the contributions of subamplitudes to the baryonia amplitude (Tables I, II, III, IV). In conclusion, the status of the considered model is discussed.

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## II. SIX-QUARK AMPLITUDES OF THE BARYONIA.

As explained in the previous paper [21] the relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting quarks. The pair quarks amplitudes  $qq \rightarrow qq$  are calculated in the framework of the dispersion  $N/D$  method with the input four-fermion interaction [22, 23] with quantum numbers of the gluon [24, 25].

The construction of the approximate solution is based on extraction of the leading singularities are close to the region  $s_{ik} \approx 4m^2$ . Such a classification of singularities makes it possible to search for an approximate solution of equations, taking into account a definite number of leading singularities and neglecting the weaker ones.

We derive the relativistic six-quark equations in the framework of the dispersion relation technique. We use only planar diagrams; the other diagrams due to the rules of  $1/N_c$  expansion [26–28] are neglected. The current generates a six-quark system. The correct equations for the amplitude are obtained by taking into account all possible subamplitudes. It corresponds to the division of complete system into subsystems with a smaller number of particles. Then one should represent a six-particle amplitude as a sum of 15 subamplitudes:

$$A = \sum_{\substack{i < j \\ i, j=1}}^6 A_{ij}. \quad (1)$$

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams. We need to consider only one group of diagrams and the amplitude corresponding to them, for example  $A_{12}$ . We shall consider the derivation of the relativistic generalization of the Faddeev-Yakubovsky approach.

In our case the low-lying baryonia are considered. We take into account the pairwise interaction of all quarks and antiquarks in the baryonia.

The system of graphical equations Fig. 1 is determined using the selfconsistent method. The coefficients are determined by the permutation of quarks [29, 30]. We should discuss the coefficient multiplying of the diagrams in the equations of Fig. 1. For example, we consider the first subamplitude  $A_1^{uu}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$ . In the Fig. 1 the first coefficient is equal to 2 (permutation particles 1 and 2). The second coefficient is equal to 6 = 3 (permutation particles 1 and 2)  $\times$  3 (we consider the third, the fifth, the sixth particles). The similar approach allows us to take into account the coefficients in all equations.

In order to represent the subamplitudes  $A_1^{uu}$ ,  $A_1^{u\bar{d}}$ ,  $A_1^{\bar{d}\bar{d}}$ ,  $A_2^{uu1\bar{d}\bar{d}}$ ,  $A_3^{uu1u\bar{d}1\bar{d}\bar{d}}$  in the form of dispersion relations, it is necessary to define the amplitudes of  $q\bar{q}$  and  $q\bar{q}$  interactions.

We use the results of our relativistic quark model [25] and write down the pair quark amplitudes in the form:

$$a_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})}, \quad (2)$$

$$B_n(s_{ik}) = \int_{(m_i+m_k)^2}^{\frac{(m_i+m_k)^2\Lambda}{4}} \frac{ds'_{ik}}{\pi} \frac{\rho_n(s'_{ik})G_n^2(s'_{ik})}{s'_{ik} - s_{ik}}. \quad (3)$$

In the case of the  $s$ -channel amplitudes we use the matrix element:  $(\bar{q}_c^a O^i q^{a'}) (\bar{q}^{b'} O^i q_c^b)$ , where  $q_c = \bar{q}C$  is the charge-conjugated spinor.  $O^i$  are operators of different types of the four-fermion interaction ( $i = S, V, T, A, P$ ),  $a, b, a', b'$  flavor indices.

Here  $G_n(s_{ik})$  are the diquark vertex functions (Table V). The vertex functions are determined by the contribution of the crossing channels. The vertex functions satisfy the Fierz relations. Since the vertex functions depend only slightly on energy it is possible to treat them as constants in our approximation and determine  $G^2 = N$  of  $N/D$  method. These vertex functions are generated from gluon coupling constant  $g$ .  $B_n(s_{ik})$  and  $\rho_n(s_{ik})$  are the Chew-Mandelstam functions with cutoff  $\Lambda$  [31] and the phase spaces, respectively:

$$\begin{aligned} \rho_n(s_{ik}, J^{PC}) = & \left( \alpha(n, J^{PC}) \frac{s_{ik}}{(m_i + m_k)^2} + \beta(n, J^{PC}) + \delta(n, J^{PC}) \frac{(m_i - m_k)^2}{s_{ik}} \right) \\ & \times \frac{\sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)}}{s_{ik}}. \end{aligned} \quad (4)$$

The coefficients  $\alpha(n, J^{PC})$ ,  $\beta(n, J^{PC})$  and  $\delta(n, J^{PC})$  are given in Table V.

Here  $n = 1$  corresponds to  $q\bar{q}$ -pairs with  $J^P = 0^-$ ,  $n = 2$  corresponds to the  $q\bar{q}$ -pairs with  $J^P = 1^-$ ,  $n = 3$  defines the  $qq$ -pairs with  $J^P = 0^+$ ,  $n = 4$  corresponds to  $J^P = 1^+$   $qq$ -pairs.

Let us extract two- and three-particle singularities in the amplitudes  $A_1^{1^{uu}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$ ,  $A_1^{1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$ ,  $A_1^{1^{u\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$ ,  $A_2^{1^{uu}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})$ ,  $A_3^{1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{12}, s_{34}, s_{56})$ :

$$A_1^{1^{uu}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1^{uu}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})B_{1^{uu}}(s_{12})}{[1 - B_{1^{uu}}(s_{12})]}, \quad (5)$$

$$A_1^{1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})B_{1^{\bar{d}\bar{d}}}(s_{12})}{[1 - B_{1^{\bar{d}\bar{d}}}(s_{12})]}, \quad (6)$$

$$A_1^{1^{u\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1^{u\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})B_{1^{u\bar{d}}}(s_{12})}{[1 - B_{1^{u\bar{d}}}(s_{12})]}, \quad (7)$$

$$A_2^{1^{uu}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) = \frac{\alpha_2^{1^{uu}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})B_{1^{uu}}(s_{12})B_{1^{\bar{d}\bar{d}}}(s_{34})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{1^{\bar{d}\bar{d}}}(s_{34})]}, \quad (8)$$

$$A_3^{1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{12}, s_{34}, s_{56}) = \frac{\alpha_3^{1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{12}, s_{34}, s_{56})B_{1^{uu}}(s_{12})B_{1^{u\bar{d}}}(s_{34})B_{1^{\bar{d}\bar{d}}}(s_{56})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{1^{u\bar{d}}}(s_{34})][1 - B_{1^{\bar{d}\bar{d}}}(s_{56})]}. \quad (9)$$

We do not extract four-particles singularities, because they are weaker than two- and three-particle singularities.

We used the classification of singularities, which was proposed in paper [32]. The construction of the approximate solution of Eqs. (5) – (9) is based on the extraction of the leading singularities of the amplitudes. The main singularities in  $s_{ik} = (m_i + m_k)^2$  are from pair rescattering of the particles  $i$  and  $k$ . First of all there are threshold square-root singularities. Also possible are pole singularities which correspond to the bound states. The diagrams of Fig. 1 apart from two-particle singularities have triangular singularities and the singularities defining the interactions of four, five and six particles. Such classification allows us to search the corresponding solution of equations by taking into account some definite number of leading singularities and neglecting all the weaker ones. We consider the approximation which defines two-particle, triangle and four-, five- and six-particle singularities. The contribution of two-particle and triangle singularities are more important, but we must take into account also the other singularities.

The five functions  $\alpha_i$  are the smooth functions of  $s_{ik}$ ,  $s_{ijk}$ ,  $s_{ijkl}$ ,  $s_{ijklm}$  as compared with the singular part of the amplitudes, hence they can be expanded in a series in the singularity point and only the first term of this series should be employed further. Using this classification, one defines the reduced amplitudes  $\alpha_i$  as well as the  $B$ -functions in the middle point of physical region of Dalitz-plot at the point  $s_0$ :

$$s_0 = \frac{s + 4 \sum_{i=1}^6 m_i^2}{\sum_{\substack{i,k=1 \\ i < k}}^6 m_{ik}^2}, \quad (10)$$

$$s_{123} = s_0 \sum_{\substack{i,k=1 \\ i < k}}^3 m_{ik}^2 - \sum_{i=1}^3 m_i^2, \quad (11)$$

$$s_{1234} = s_0 \sum_{\substack{i,k=1 \\ i < k}}^4 m_{ik}^2 - 2 \sum_{i=1}^4 m_i^2. \quad (12)$$

Such choice of point  $s_0$  allows us to replace integral equations (Fig. 1) by the algebraic equations (13) – (17), respectively:

$$\alpha_1^{1^{uu}} = \lambda + 2I_1(1^{uu}1^{uu})\alpha_1^{1^{uu}} + 6I_1(1^{uu}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}}, \quad (13)$$

$$\alpha_1^{1^{\bar{d}\bar{d}}} = \lambda + 2I_1(1^{\bar{d}\bar{d}}1^{\bar{d}\bar{d}})\alpha_1^{1^{\bar{d}\bar{d}}} + 6I_1(1^{\bar{d}\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}}, \quad (14)$$

$$\alpha_1^{1^{u\bar{d}}} = \lambda + 2I_1(1^{u\bar{d}}1^{uu})\alpha_1^{1^{uu}} + 2I_1(1^{u\bar{d}}1^{\bar{d}\bar{d}})\alpha_1^{1^{\bar{d}\bar{d}}} + 4I_1(1^{u\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} + 4I_2(1^{u\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_2^{1^{uu}1^{\bar{d}\bar{d}}}, \quad (15)$$

$$\begin{aligned} \alpha_2^{1^{uu}1^{\bar{d}\bar{d}}} &= \lambda + 2I_4(1^{uu}1^{\bar{d}\bar{d}}1^{uu})\alpha_1^{1^{uu}} + 2I_4(1^{\bar{d}\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_1^{1^{\bar{d}\bar{d}}} + 4I_3(1^{uu}1^{\bar{d}\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} \\ &+ 4I_6(1^{uu}1^{\bar{d}\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_2^{1^{uu}1^{\bar{d}\bar{d}}} + 4I_8(1^{uu}1^{\bar{d}\bar{d}}1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}})\alpha_3^{1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}}, \end{aligned} \quad (16)$$

$$\begin{aligned} \alpha_3^{1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}} &= \lambda + 2I_9(1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}1^{uu})\alpha_1^{1^{uu}} + 2I_9(1^{\bar{d}\bar{d}}1^{u\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_1^{1^{\bar{d}\bar{d}}} + 2I_9(1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} \\ &+ 4I_9(1^{uu}1^{\bar{d}\bar{d}}1^{u\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} + 2I_9(1^{\bar{d}\bar{d}}1^{u\bar{d}}1^{uu}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} + 4I_{10}(1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_2^{1^{uu}1^{\bar{d}\bar{d}}}, \end{aligned} \quad (17)$$

where  $\lambda_i$  are the current constants. We used the functions  $I_1, I_2, I_3, I_4, I_6, I_8, I_9, I_{10}$ :

$$I_1(ij) = \frac{B_j(s_0^{13})}{B_i(s_0^{12})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \int_{-1}^{+1} \frac{dz_1(1)}{2} \frac{1}{1 - B_j(s'_{13})}, \quad (18)$$

$$\begin{aligned} I_2(ijk) &= \frac{B_j(s_0^{13})B_k(s_0^{24})}{B_i(s_0^{12})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(2)}{2} \int_{-1}^{+1} \frac{dz_2(2)}{2} \\ &\times \int_{z_3(2)^-}^{z_3(2)^+} dz_3(2) \frac{1}{\sqrt{1 - z_1^2(2) - z_2^2(2) - z_3^2(2) + 2z_1(2)z_2(2)z_3(2)}} \\ &\times \frac{1}{1 - B_j(s'_{13})} \frac{1}{1 - B_k(s'_{24})}, \end{aligned} \quad (19)$$

$$\begin{aligned} I_3(ijk) &= \frac{B_k(s_0^{23})}{B_i(s_0^{12})B_j(s_0^{34})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\ &\times \int_{(m_3+m_4)^2}^{\frac{(m_3+m_4)^2\Lambda_j}{4}} \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34})\rho_j(s'_{34})}{s'_{34} - s_0^{34}} \int_{-1}^{+1} \frac{dz_1(3)}{2} \int_{-1}^{+1} \frac{dz_2(3)}{2} \frac{1}{1 - B_k(s'_{23})}, \end{aligned} \quad (20)$$

$$I_4(ijk) = I_1(ik), \quad (21)$$

$$I_6(ijkl) = I_1(ik) \cdot I_1(jl), \quad (22)$$

$$I_8(ijklm) = \frac{B_k(s_0^{15})B_l(s_0^{23})B_m(s_0^{46})}{B_i(s_0^{12})B_j(s_0^{34})} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2\Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}}$$

$$\begin{aligned}
& \times \frac{(m_3+m_4)^2 \Lambda_j}{(m_3+m_4)^2} \int_{-1}^4 \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34}) \rho_j(s'_{34})}{s'_{34} - s_0^{34}} \\
& \times \frac{1}{(2\pi)^2} \int_{-1}^{+1} \frac{dz_1(8)}{2} \int_{-1}^{+1} \frac{dz_2(8)}{2} \int_{-1}^{+1} \frac{dz_3(8)}{2} \int_{z_4(8)^-}^{z_4(8)^+} dz_4(8) \int_{-1}^{+1} \frac{dz_5(8)}{2} \int_{z_6(8)^-}^{z_6(8)^+} dz_6(8) \\
& \times \frac{1}{\sqrt{1 - z_1^2(8) - z_3^2(8) - z_4^2(8) + 2z_1(8)z_3(8)z_4(8)}} \\
& \times \frac{1}{\sqrt{1 - z_2^2(8) - z_5^2(8) - z_6^2(8) + 2z_2(8)z_5(8)z_6(8)}} \\
& \times \frac{1}{1 - B_k(s'_{15})} \frac{1}{1 - B_l(s'_{23})} \frac{1}{1 - B_m(s'_{46})}, \tag{23}
\end{aligned}$$

$$I_9(ijkl) = I_3(ijl), \tag{24}$$

$$\begin{aligned}
I_{10}(ijklm) &= \frac{B_l(s_0^{23})B_m(s_0^{45})}{B_i(s_0^{12})B_j(s_0^{34})B_k(s_0^{56})} \frac{(m_1+m_2)^2 \Lambda_i}{(m_1+m_2)^2} \int_{-1}^4 \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12}) \rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\
& \times \frac{(m_3+m_4)^2 \Lambda_j}{(m_3+m_4)^2} \int_{-1}^4 \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34}) \rho_j(s'_{34})}{s'_{34} - s_0^{34}} \frac{(m_5+m_6)^2 \Lambda_k}{(m_5+m_6)^2} \int_{-1}^4 \frac{ds'_{56}}{\pi} \frac{G_k^2(s_0^{56}) \rho_k(s'_{56})}{s'_{56} - s_0^{56}} \\
& \times \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(10)}{2} \int_{-1}^{+1} \frac{dz_2(10)}{2} \int_{-1}^{+1} \frac{dz_3(10)}{2} \int_{-1}^{+1} \frac{dz_4(10)}{2} \int_{z_5(10)^-}^{z_5(10)^+} dz_5(10) \\
& \times \frac{1}{\sqrt{1 - z_1^2(10) - z_4^2(10) - z_5^2(10) + 2z_1(10)z_4(10)z_5(10)}} \\
& \times \frac{1}{1 - B_l(s'_{23})} \frac{1}{1 - B_m(s'_{45})}, \tag{25}
\end{aligned}$$

where  $i, j, k, l, m$  correspond to the diquarks with the spin-parity  $J^P = 0^+, 1^+$  and mesons with the spin-parity  $J^P = 0^-, 1^-$ .

The other choices of point  $s_0$  do not change essentially the contributions of  $\alpha_i$ , therefore we omit the indices  $s_0^{ik}$ . Since the vertex functions depend only slightly on energy, it is possible to treat them as constants in our approximation.

We can pass from the integration over cosines of the angles to the integration over the subenergies [21]. In the relativistic invariant solution the center of mass of particles 1, 2 by the standard method is treated [21].

The system of graphical equations Fig. 1 is determined by the subamplitudes  $A_1, A_2, A_3$ .  $B\bar{B}$  states ( $A_2$ ) are constructed without the mixing of quarks and antiquarks. Therefore we did not use the three mesons and meson plus tetraquark states. But the subamplitudes  $A_1$  and  $A_3$  contain the quark-antiquark pairs. Then the algebraic equations (13) – (17) take into account the contributions reduced amplitudes  $\alpha_1^{1uu}, \alpha_1^{1\bar{d}\bar{d}}, \alpha_1^{1u\bar{d}}, \alpha_2^{1uu}1^{\bar{d}\bar{d}}, \alpha_3^{1uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}$ . The solution of the system of equations are considered as:

$$\alpha_i(s) = F_i(s, \lambda_i)/D(s), \tag{26}$$

where zeros of  $D(s)$  determinants define the masses of bound states of baryonia.

We have analyzed in the subamplitudes of a quark and an antiquark and did not obtain the bound state with the model parameters.

As example, we consider the equations for the quark content  $uuu\bar{d}\bar{d}\bar{d}$  with the isospin  $I = 3$  and the spin-parity

$J^P = 3^-$  (Fig. 1). The similar equations have been calculated for the isospin  $I = 0, 1, 2, 3$  and the spin-parity  $J^P = 0^-, 1^-, 2^-, 3^-$ . We take into account the  $u$  and  $d$  quarks.

The functions  $I_1, I_2, I_3, I_4, I_6, I_8, I_9, I_{10}$  determine the interaction of the quarks and the antiquarks. These functions take into account the contributions of the Chew-Mandelstam functions, which are constructed in the model for the quark-antiquark pairs with various quantum numbers using the unitarity condition.

### III. CALCULATION RESULTS.

The poles of the reduced amplitudes  $\alpha_i$  correspond to the bound states and determine the masses of the baryonia. The dynamical mixing between the subamplitudes of baryonia is considered. We derived the relativistic six-quark equations in the framework of the dispersion technique. The pair quarks amplitudes  $qq \rightarrow qq$  and  $q\bar{q} \rightarrow q\bar{q}$  are calculated with the dispersion  $N/D$  method using the input four-fermion interaction [22, 23] with the quantum numbers of the gluon [24, 25].

The model under consideration proceeds from the assumption that the confinement radius is sufficiently larger than constituent quark radii as well as the radii of the forces which bound the low-lying hadrons. It means that quark interaction forces are the two component ones. The long-range component is due to the confinement. In the present paper, when the low-lying hadrons are considered, the long-range component of the forces is neglected.

We manage with the quarks as with real particles. However, in the soft region, the quark diagrams should be treated as spectral integrals over quark masses with the spectral density  $\rho(m^2)$ : the integration over quark masses in the amplitudes puts away the quark singularities and introduces the hadron ones. We can believe that the approximation  $\rho(m^2) \rightarrow \delta(m^2 - m_q^2)$  could be possible for the low-lying hadrons. We hope that this approach is sufficiently good for the calculation of the low-lying baryonia being carried out here. The problem of the distribution over quark masses is important when one considers the high-excited states.

The four-quark interaction is considered as an input:

$$g_V (\bar{q}\lambda I_f \gamma_\mu q)^2. \quad (27)$$

Here  $I_f$  is the unity matrix in the flavor space ( $u, d$ ).  $\lambda$  are the color Gell-Mann matrices.

We introduce the scale of the dimensional parameters [25]:

$$g = \frac{m^2}{\pi^2} g_V, \quad \Lambda = \frac{4\Lambda(ik)}{(m_i + m_k)^2}. \quad (28)$$

Here  $m_i$  and  $m_k$  are the quark masses in the intermediate state of the quark loop. Dimensionless parameters  $g$  and  $\Lambda$  are supposed to be the constants which are independent of the quark interaction type. In the case under question the interacting pairs of particles do not form the bound states. The attraction of quark-antiquark and quark-quark pairs is not enough for the construction of the bound state. This is similar to the case of the four-quark systems [32]. Therefore, the integration in the dispersion integral run from  $4m^2$  to  $\Lambda$ .

The quark masses of model  $m_{u,d} = 410 \text{ MeV}$  coincide with ordinary baryon ones [21]. The model in question has only two parameters: the cutoff parameter  $\Lambda = 11$  and the gluon coupling constant  $g = 0.314$ . These parameters are similar to the previous paper [21] ones.

The estimation of theoretical error on the baryonia masses is  $1 \text{ MeV}$ . This result was obtained by the choice of model parameters.

Some authors had investigated the tetraquark system with the three-body  $qq\bar{q}$  and  $q\bar{q}\bar{q}$  interaction, whose existence has no direct effect on the ordinary hadron states [33, 34]. But assuming the most general two-body quark Hamiltonian [35, 36], Pirjol and Schat derive universal correlations among masses and mixing angles which are valid in any model for quark interactions containing only two-body interactions. Deviation from these predictions provide no evidence for the presence of spin-flavor dependent three-body quark interactions.

Our model is based on the three principles of unitarity, analyticity and crossing symmetry. The principle of unitarity are applied to the two-body subenergy channels.

Further experimental and theoretical efforts are required in order to satisfactory explain the presence of three-body quark interactions.

In the Table I the calculated masses of nonstrange baryonia are shown. The contributions of subamplitudes to the six-quark amplitude are represented in the Tables II, III, IV. The states  $(\Delta\bar{\Delta} + \Delta\bar{n} + n\bar{\Delta} + n\bar{n})$ ,  $(\Delta\bar{\Delta} + \Delta\bar{p} + p\bar{\Delta} + p\bar{p})$  and  $(\Delta\bar{\Delta} + \Delta\bar{n} + p\bar{\Delta} + p\bar{n})$  with the isospins  $I = 0, 1$  and the spin-parity  $J^P = 0^-$  possess the mass  $M = 1835 \text{ MeV}$ . We predict the degeneracy of the some states. For the  $(\Delta\bar{\Delta} + p\bar{\Delta})$ ,  $(\Delta\bar{\Delta} + \Delta\bar{n})$ ,  $(\Delta\bar{\Delta} + n\bar{\Delta})$  and  $(\Delta\bar{\Delta} + \Delta\bar{p})$  with

the isospins  $I = 1, 2$  and the spin-parity  $J^P = 0^-$  the mass  $M = 1928 \text{ MeV}$  is obtained. The low-lying state  $uuu\bar{d}\bar{d}\bar{d}$  ( $\Delta\bar{\Delta}$ ) with the isospin  $I = 3$  and the spin-parity  $J^P = 1^-$  possesses the mass  $M = 1783 \text{ MeV}$ . The dynamical mixing between the five subamplitudes (similar to the Fig. 1) is considered. Therefore the multiquark state will be stable. The states  $uuu\bar{u}\bar{u}\bar{u}$  ( $\Delta\bar{\Delta}$ ) and  $ddd\bar{d}\bar{d}\bar{d}$  ( $\Delta\bar{\Delta}$ ) with the isospin  $I = 0$  and the spin-parity  $J^P = 0^-$  have the mass  $M = 1973 \text{ MeV}$ .

We predict the degeneracy of baryonia  $M(uud\bar{d}\bar{d}\bar{d}, I = 2) = M(uuu\bar{d}\bar{d}\bar{d}, I = 2) = M(udd\bar{d}\bar{d}\bar{d}, I = 1) = M(uuu\bar{u}\bar{u}\bar{d}, I = 1)$ . For the states  $M(ud\bar{u}\bar{d}\bar{d}, I = 1) = M(udd\bar{u}\bar{d}\bar{d}, I = 0) = M(uud\bar{u}\bar{u}\bar{d}, I = 0)$  and  $M(uuu\bar{u}\bar{u}\bar{u}, I = 0) = M(ddd\bar{d}\bar{d}\bar{d}, I = 0)$  the degeneracy is also obtained.

A somewhat simple picture of baryonium is that of a deuteron-like  $N\bar{N}$  bound state or resonance, benefiting from the attractive potential mediated by the exchange of gluon [21]. We consider the influence of the contributions of quark-antiquark pairs.

Entem and Fernandez, describing scattering data and mass shifts of  $p\bar{p}$  levels in a constituent quark model, assign the threshold enhancement to final-state interaction [37, 38]. Zou and Chiang find that final state interaction makes an important contribution to the  $p\bar{p}$  near threshold enhancement [39].

BES collaboration measured the mass  $X(1835)$  to be  $M = 1833.7 \text{ MeV}$  and its width to be  $\Gamma = 67.7 \text{ MeV}$  [2].

The state with  $M = 1835 \text{ MeV}$  is considered as  $p\bar{p}$  state [4] or the second radial excitation of  $\eta'$  meson [40].

In our case this state have following content  $\Delta\bar{\Delta} + \Delta\bar{p} + p\bar{\Delta} + p\bar{p}$  with isospin  $I = 0$  and spin-parity  $J^P = 0^-$ .

We calculated the masses of baryonia with the isospin  $I = 0, 1, 2, 3$  and spin-parity  $J^P = 0^-, 1^-, 2^-, 3^-$  (Table I).

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TABLE I: S-wave baryonia masses. Parameters of model: cutoff  $\Lambda = 11.0$ , gluon coupling constant  $g = 0.314$ . Quark masses  $m_{u,d} = 410 \text{ MeV}$ .

$I$	Quark content (baryonia)	$J$	Mass (MeV)
0	$uuu\bar{u}\bar{u} (\Delta\bar{\Delta}),$ $ddd\bar{d}\bar{d} (\Delta\bar{\Delta})$	0	1973
		1	1824
		2	1938
		3	2290
0; 1	$udd\bar{u}\bar{d} (\Delta\bar{\Delta} + \Delta\bar{n} + n\bar{\Delta} + n\bar{n}),$ $uud\bar{u}\bar{u} (\Delta\bar{\Delta} + \Delta\bar{p} + p\bar{\Delta} + p\bar{p});$ $uud\bar{u}\bar{d} (\Delta\bar{\Delta} + \Delta\bar{n} + p\bar{\Delta} + p\bar{n})$	0	1835
		1	1784
		2	1851
		3	2455
1; 2	$udd\bar{d}\bar{d} (\Delta\bar{\Delta} + n\bar{\Delta}),$ $uuu\bar{u}\bar{d} (\Delta\bar{\Delta} + \Delta\bar{p});$ $uud\bar{d}\bar{d} (\Delta\bar{\Delta} + p\bar{\Delta}),$ $uuu\bar{u}\bar{d} (\Delta\bar{\Delta} + \Delta\bar{n})$	0	1928
		1	1770
		2	1857
		3	2395
3	$uuu\bar{d}\bar{d} (\Delta\bar{\Delta})$	0	2067
		1	1783
		2	1938
		3	2290

TABLE II:  $IJ = 00$ ,  $uud\bar{u}\bar{d}$  (1835 MeV),  $\Lambda = 11.0$ ,  $g = 0.314$ .

Subamplitudes	Contributions, percent
$A_1^{uu}$	3.7
$A_1^{\bar{u}\bar{u}}$	3.7
$A_1^{u\bar{u}}$	9.0
$A_1^{u\bar{d}}$	7.8
$A_1^{d\bar{u}}$	7.8
$A_1^{d\bar{d}}$	6.7
$A_1^{ud}$	3.6
$A_1^{\bar{u}\bar{d}}$	3.6
$A_1^{u\bar{u}}$	7.0
$A_1^{0u\bar{d}}$	6.8
$A_1^{0d\bar{u}}$	6.8
$A_1^{0d\bar{d}}$	6.6
$A_2^{1uu1\bar{u}\bar{u}}$	2.4
$A_2^{1uu0\bar{u}\bar{d}}$	2.0
$A_2^{\bar{u}\bar{u}0ud}$	2.6
$A_2^{0ud0\bar{u}\bar{d}}$	2.8
$A_3^{1uu0d\bar{d}1\bar{u}\bar{u}}$	4.0
$A_3^{1uu1d\bar{u}0\bar{u}\bar{d}}$	4.5
$A_3^{\bar{u}\bar{u}1u\bar{d}0ud}$	4.5
$A_3^{0ud0u\bar{u}0\bar{u}\bar{d}}$	4.3
$\sum A_1$	73.0
$\sum A_2$	9.8
$\sum A_3$	17.2

TABLE III:  $IJ = 33$ ,  $uu\bar{d}\bar{d}$  (2290 MeV),  $\Lambda = 11.0$ ,  $g = 0.314$ .

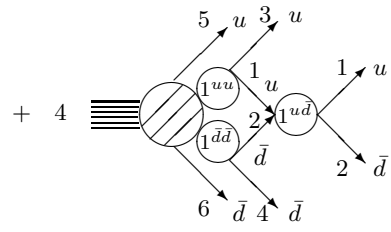
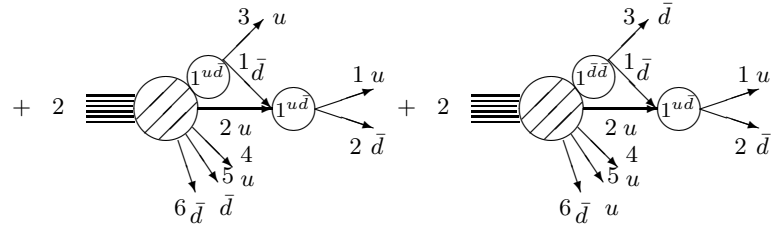
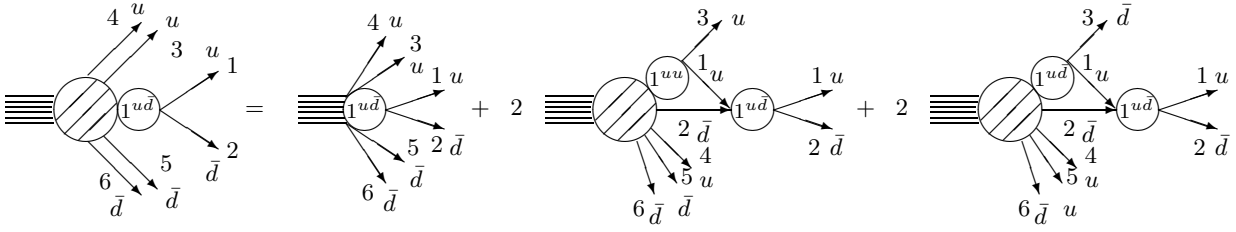
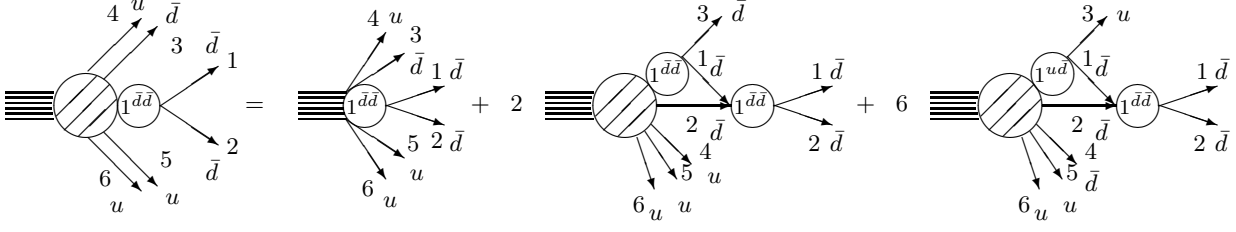
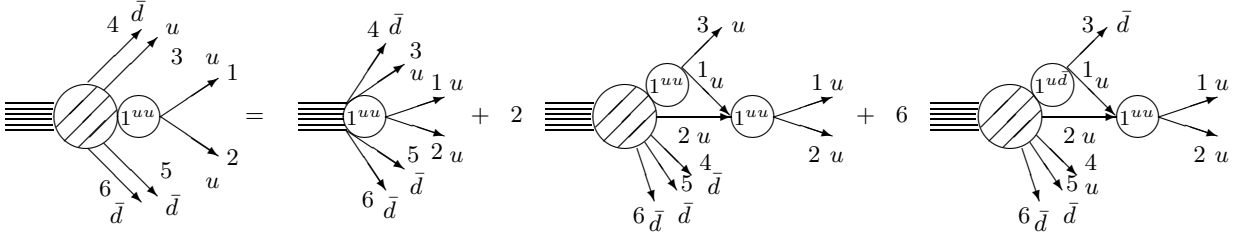
Subamplitudes	Contributions, percent
$A_1^{uu}$	9.9
$A_1^{\bar{d}\bar{d}}$	9.9
$A_1^{u\bar{d}}$	25.4
$A_2^{1uu_1\bar{d}\bar{d}}$	14.5
$A_3^{1uu_1\bar{d}\bar{d}_1u\bar{d}}$	40.3
$\sum A_1$	45.2
$\sum A_2$	14.5
$\sum A_3$	40.3

TABLE IV:  $IJ = 31$ ,  $uu\bar{d}\bar{d}$  (1783 MeV),  $\Lambda = 11.0$ ,  $g = 0.314$ .

Subamplitudes	Contributions, percent
$A_1^{uu}$	14.5
$A_1^{\bar{d}\bar{d}}$	14.5
$A_1^{u\bar{d}}$	34.8
$A_1^{0u\bar{d}}$	25.2
$A_2^{1uu_1\bar{d}\bar{d}}$	11.1
$\sum A_1$	88.9
$\sum A_2$	11.1

TABLE V: Vertex functions and Chew-Mandelstam coefficients.

$i$	$G_i^2(s_{kl})$	$\alpha_i$	$\beta_i$	$\delta_i$
$0^+$ diquark	$\frac{4g}{3} - \frac{8gm_{kl}^2}{(3s_{kl})}$	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$	0
$1^+$ diquark	$\frac{2g}{3}$	$\frac{1}{3}$	$\frac{4m_k m_l}{3(m_k + m_l)^2} - \frac{1}{6}$	$-\frac{1}{6} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$
$0^-$ meson	$\frac{8g}{3} - \frac{16gm_{kl}^2}{(3s_{kl})}$	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$	0
$1^-$ meson	$\frac{4g}{3}$	$\frac{1}{3}$	$\frac{4m_k m_l}{3(m_k + m_l)^2} - \frac{1}{6}$	$-\frac{1}{6} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$



$$\begin{aligned}
& \text{Diagram 1} = \text{Diagram 2} + 4 \text{Diagram 3} + 2 \text{Diagram 4} \\
& + 2 \text{Diagram 5} + 4 \text{Diagram 6} \\
& + 4 \text{Diagram 7}
\end{aligned}$$

The diagrams are as follows:

- Diagram 1:** A shaded circle with three horizontal lines on the left. Two circles labeled  $1^{uu}$  and  $1^{\bar{d}\bar{d}}$  are inside. Arrows point out from the right:  $5^u$ ,  $1^u$ ,  $2^u$ ,  $3^{\bar{d}}$ ,  $6^{\bar{d}}$ ,  $4^{\bar{d}}$ .
- Diagram 2:** Similar to Diagram 1, but with different internal connections.
- Diagram 3:** Similar to Diagram 1, but with different internal connections.
- Diagram 4:** Similar to Diagram 1, but with different internal connections.
- Diagram 5:** Similar to Diagram 1, but with different internal connections.
- Diagram 6:** Similar to Diagram 1, but with different internal connections.
- Diagram 7:** Similar to Diagram 1, but with different internal connections.

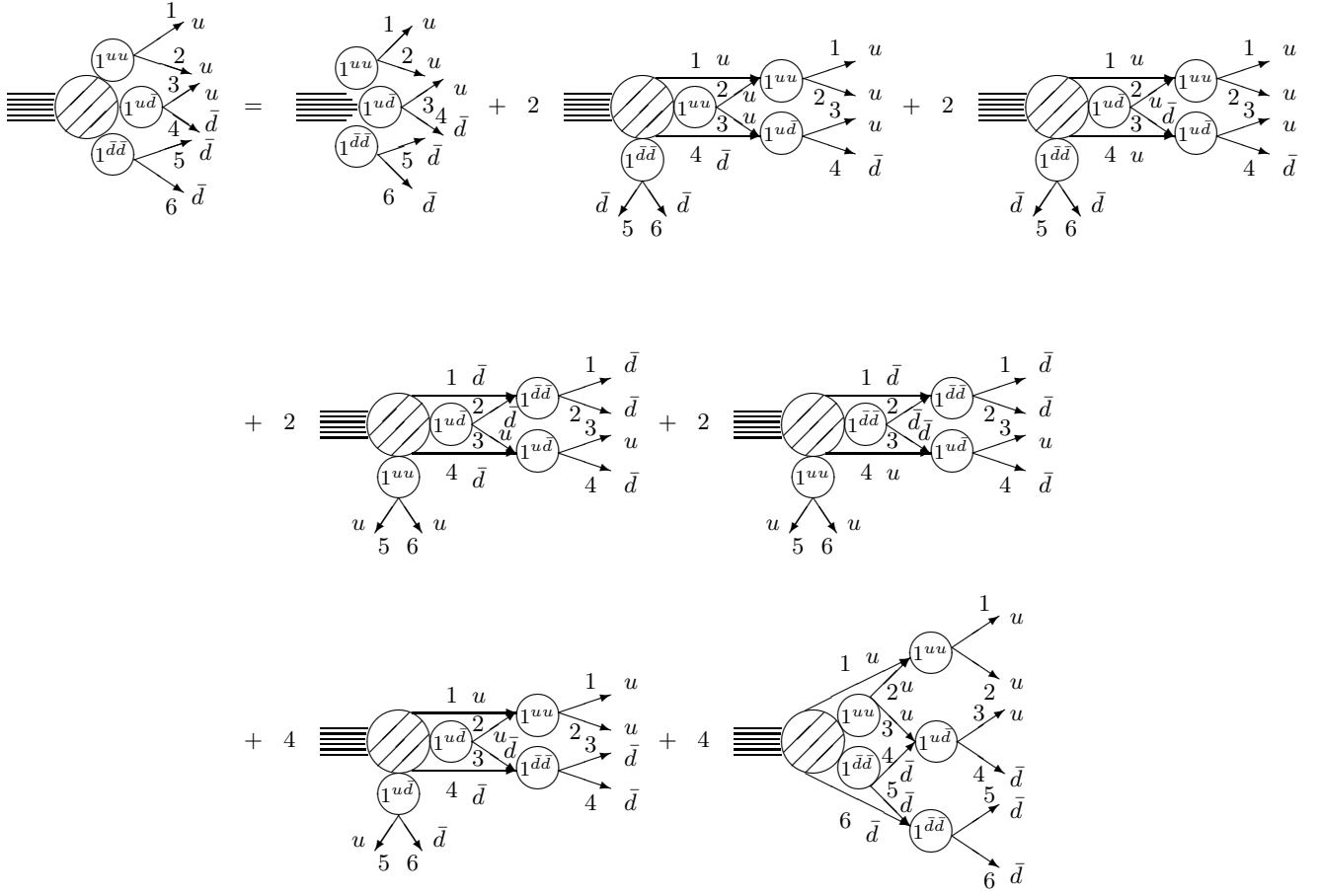


Fig. 1. Graphic representation of the equations for the six-quark subamplitudes  $A_l$  ( $l = 1, 2, 3, 4, 5$ ) in the case of baryonium  $uu\bar{d}\bar{d}\bar{d}$   $IJ = 33$ .